



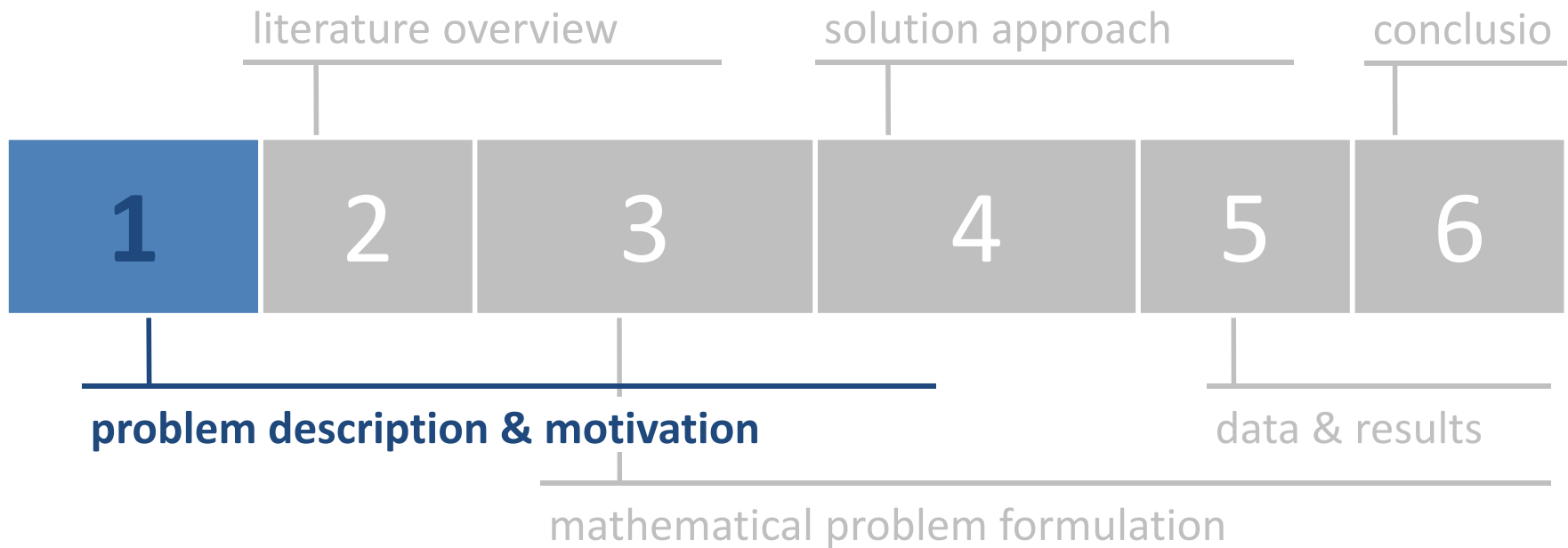
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Solving the Dynamic

**Ambulance Dispatching & Relocation Problem**

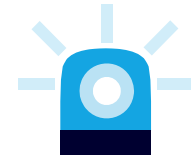
using **Approximate Dynamic Programming**

**Dr. Verena Schmid**



# problem description & motivation

- **faced by Emergency Service Providers (ESPs)**
  - manage **fleet** of ambulances
  - reach **patients** in case of emergency asap
- **fundamental decisions**
  - ambulance location
  - dispatching
  - reinsertion
  - relocate



where?

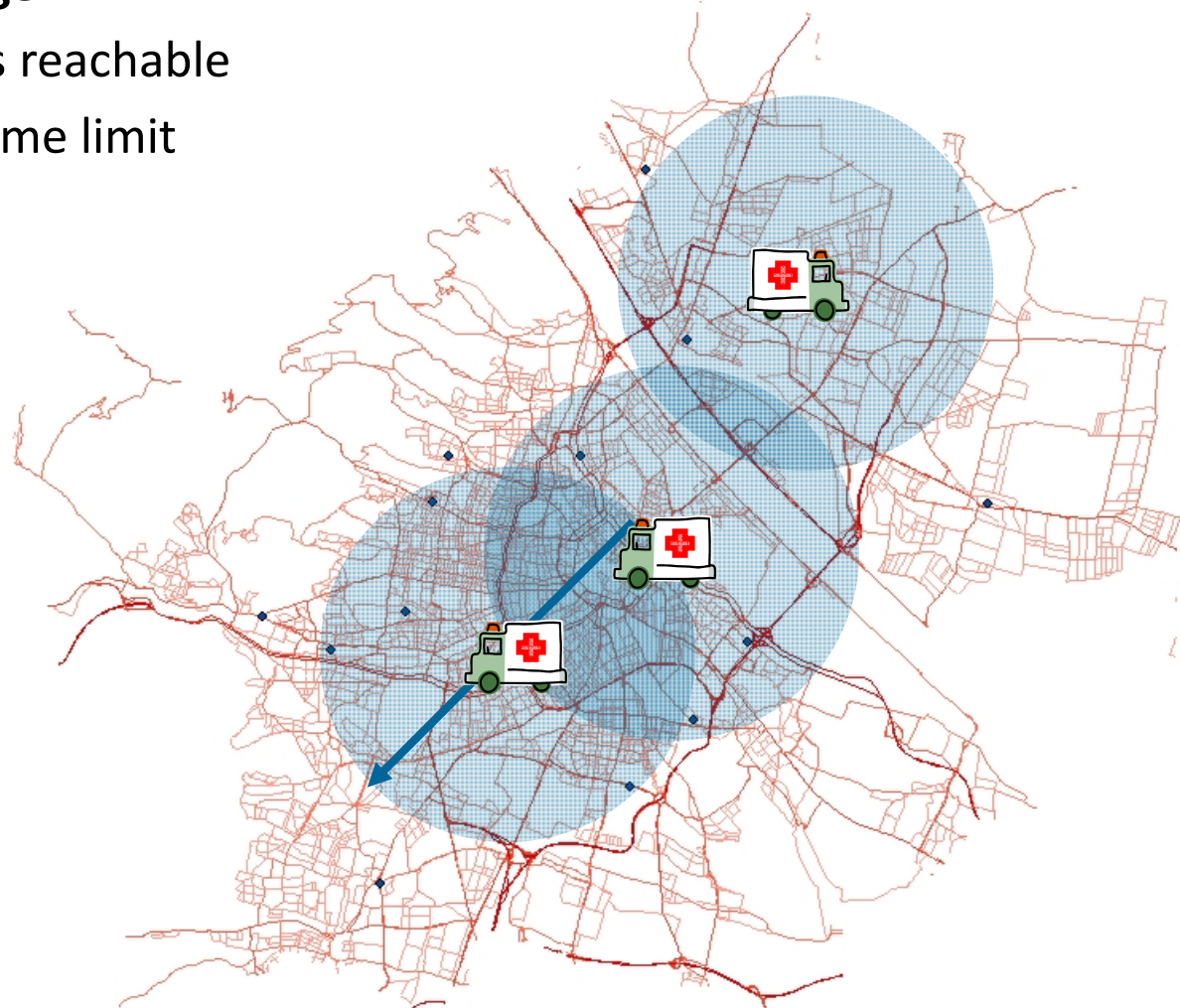
which?

where next?

where else?

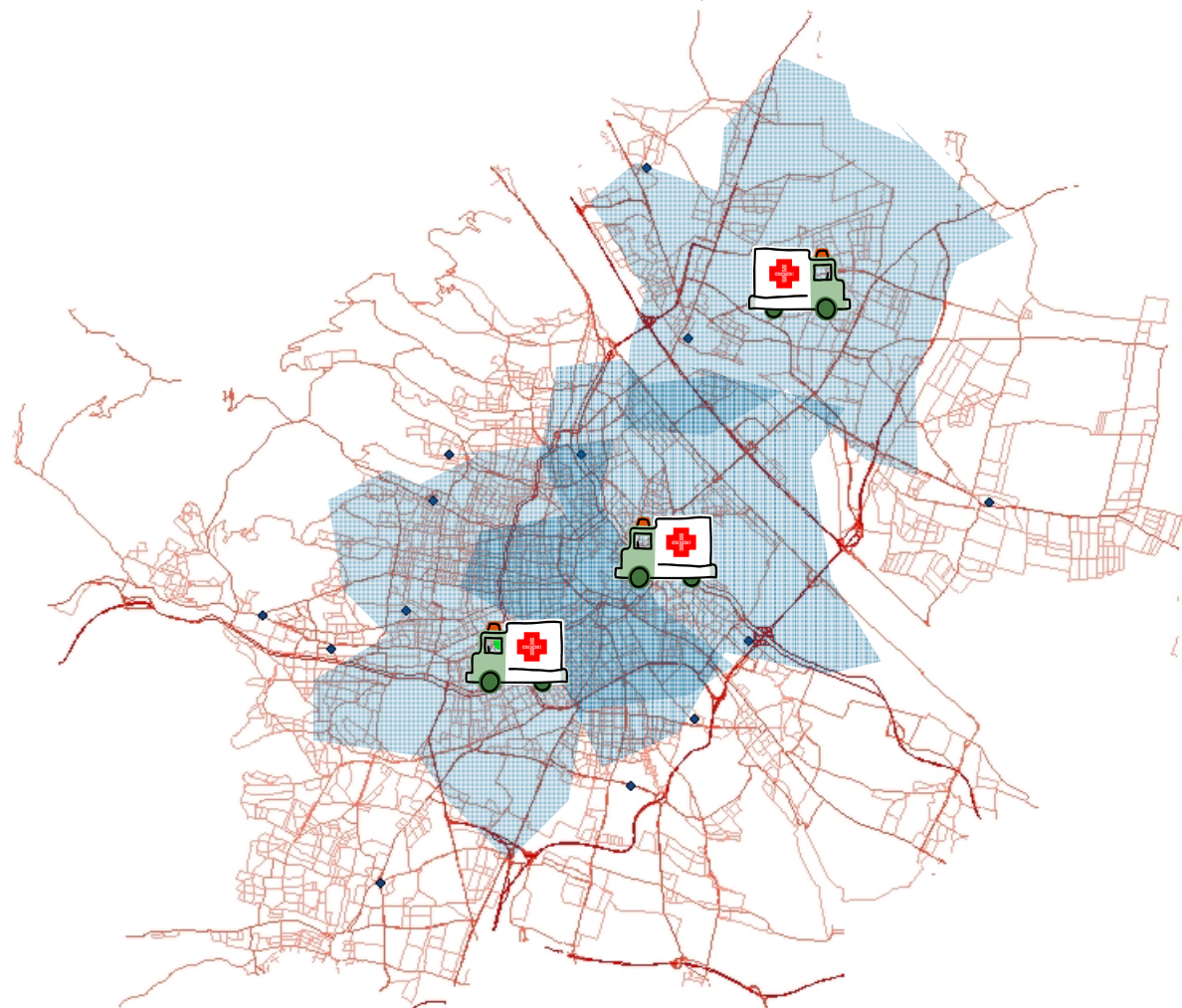
# Q1: **where** to locate ambulances?

- **optimize coverage**
  - areas/patients reachable
  - within given time limit



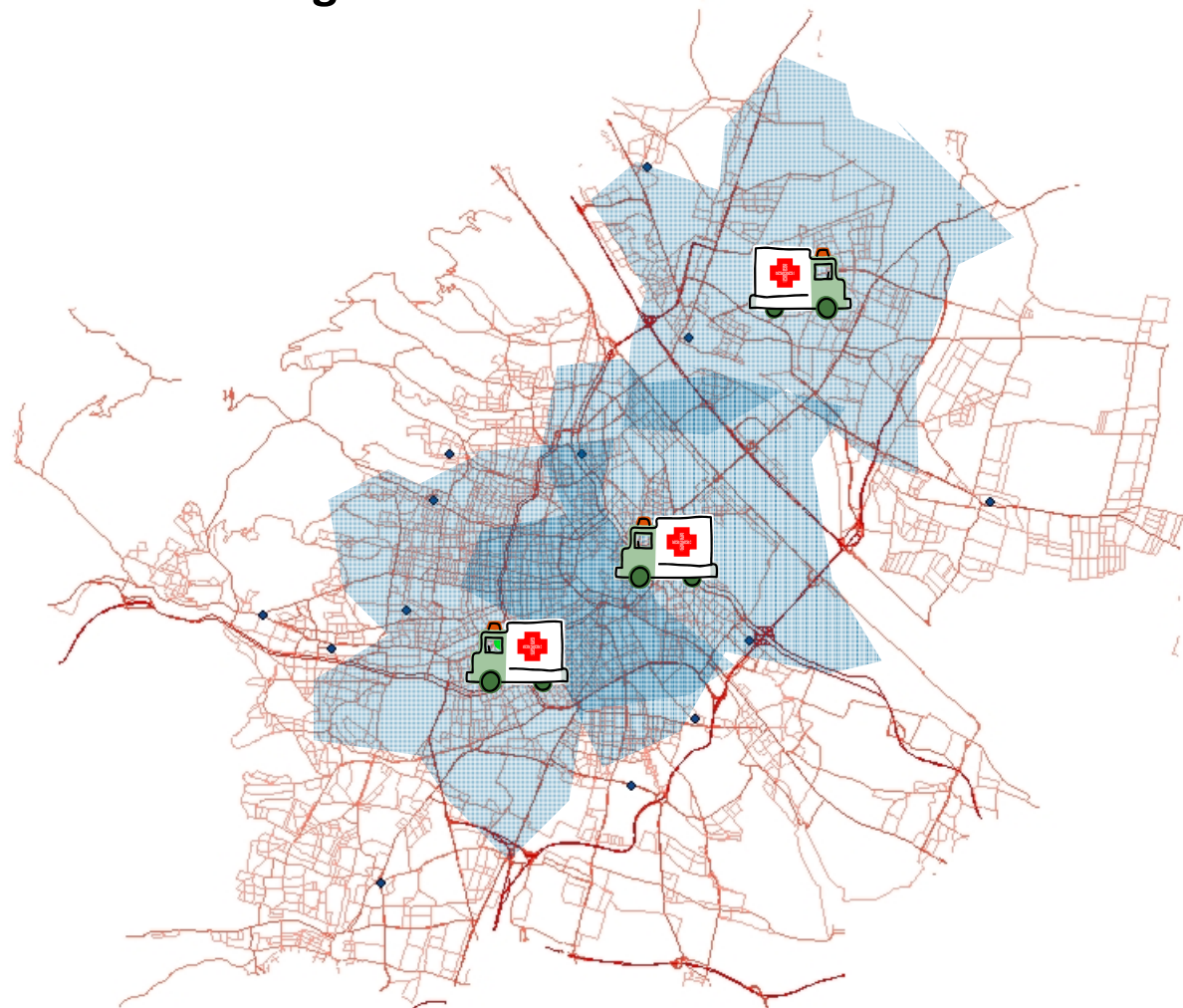
# Q1: where to locate ambulances?

- using real street network



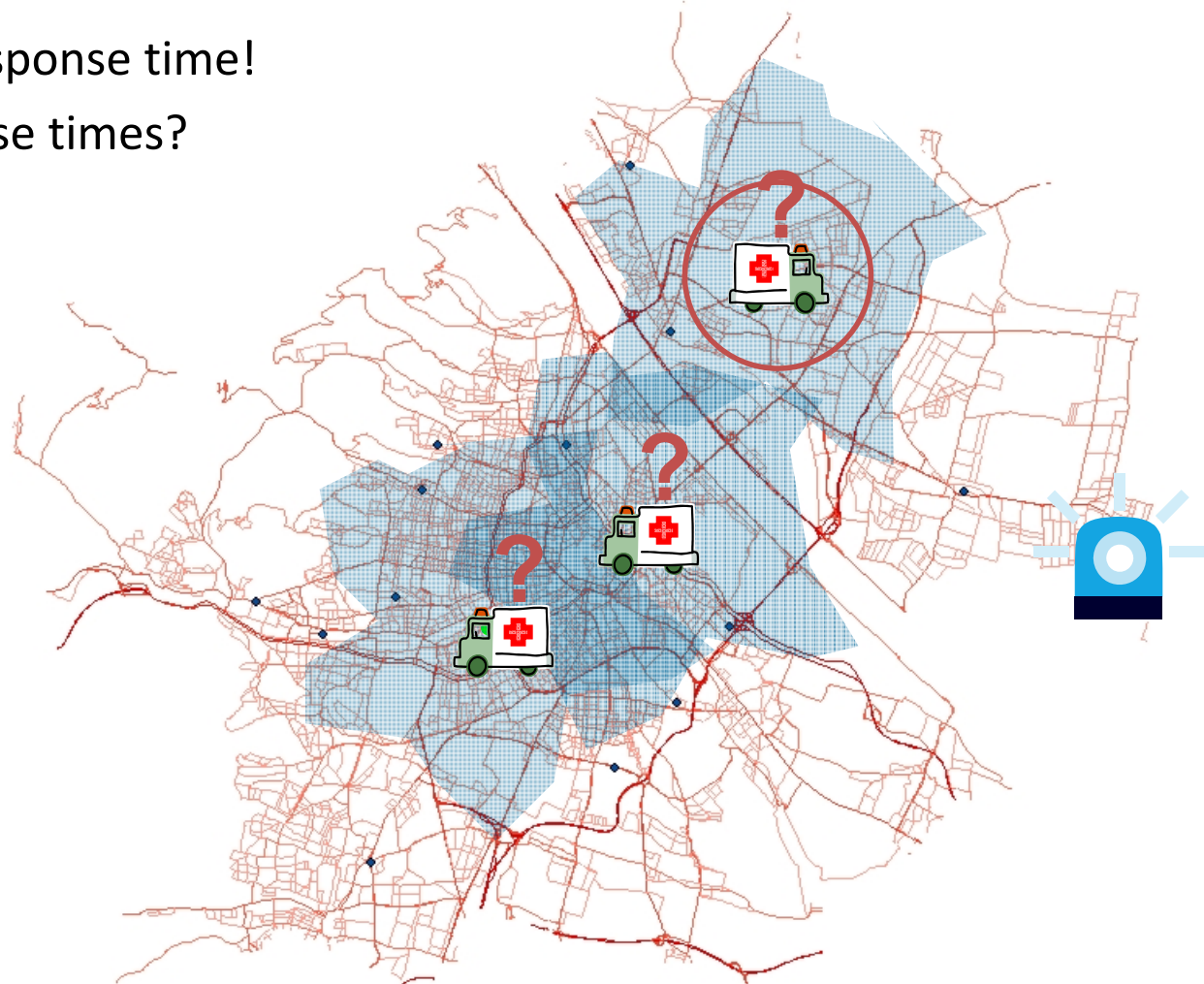
# Q1: where to locate ambulances?

- using time dependent travelling times



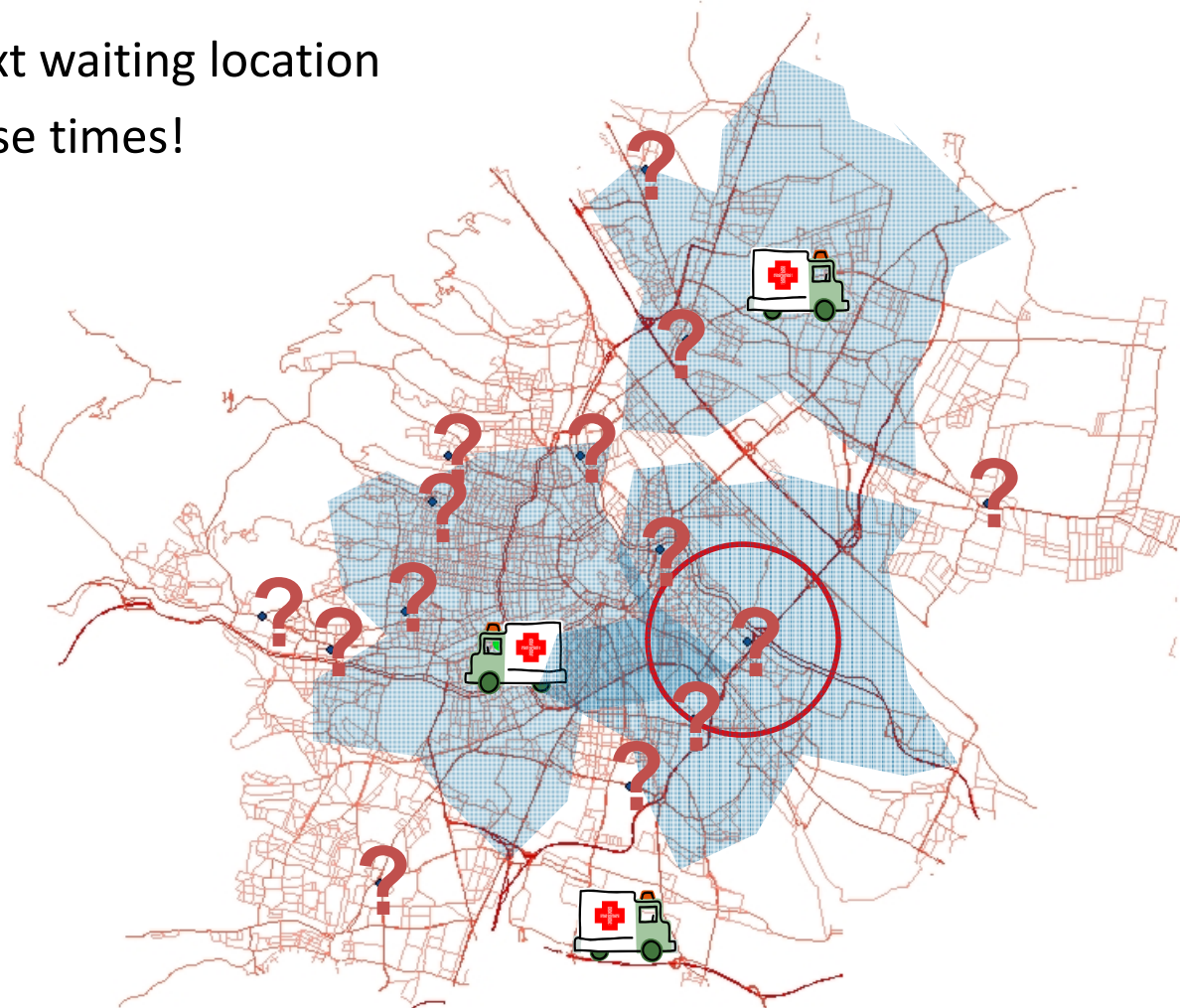
## Q2: **which** ambulance shall be sent?

- **dispatching**
  - immediate response time!
  - future response times?



## Q3: **where** to send ambulance next?

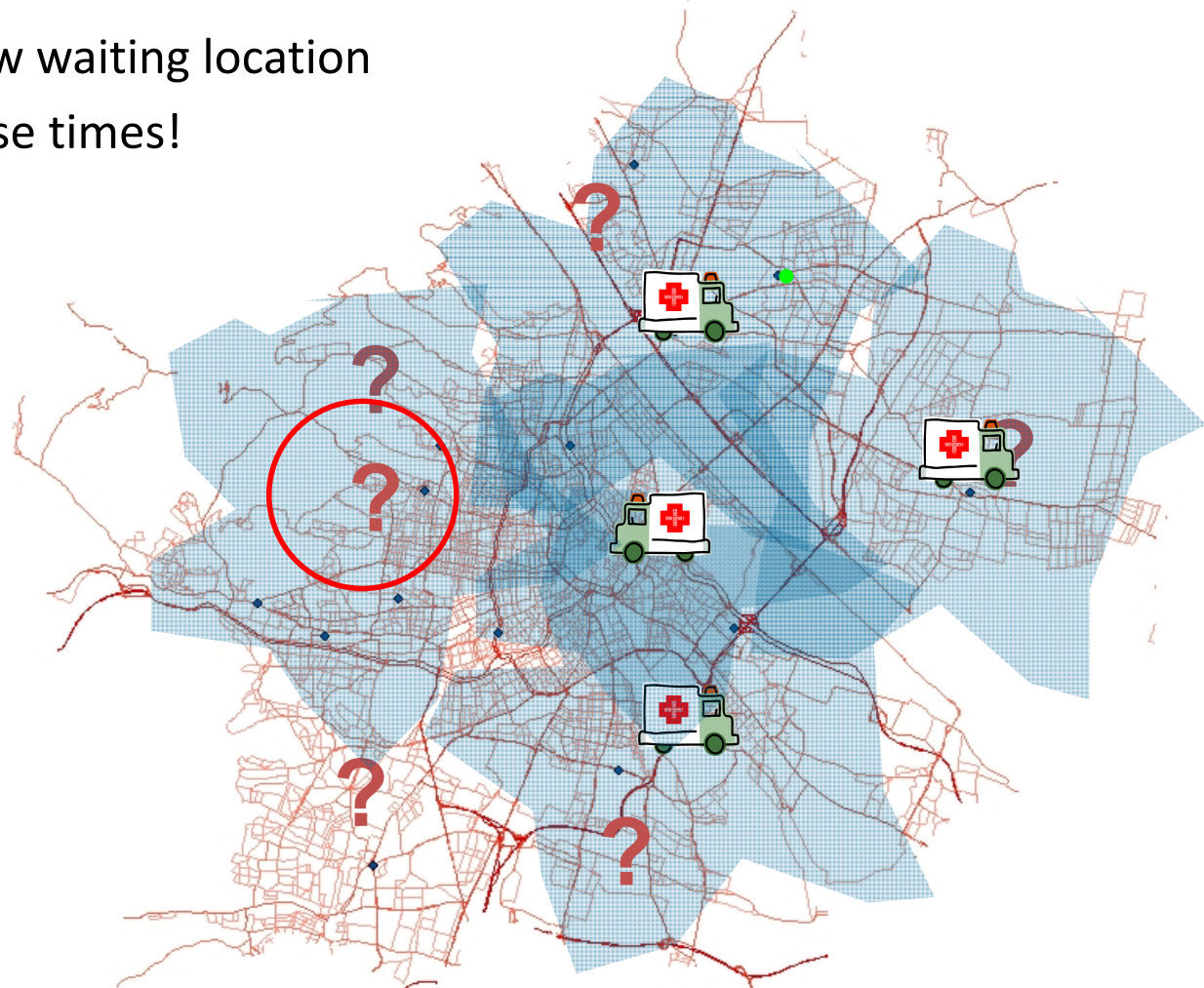
- **reinsertion**
  - determine next waiting location
  - future response times!



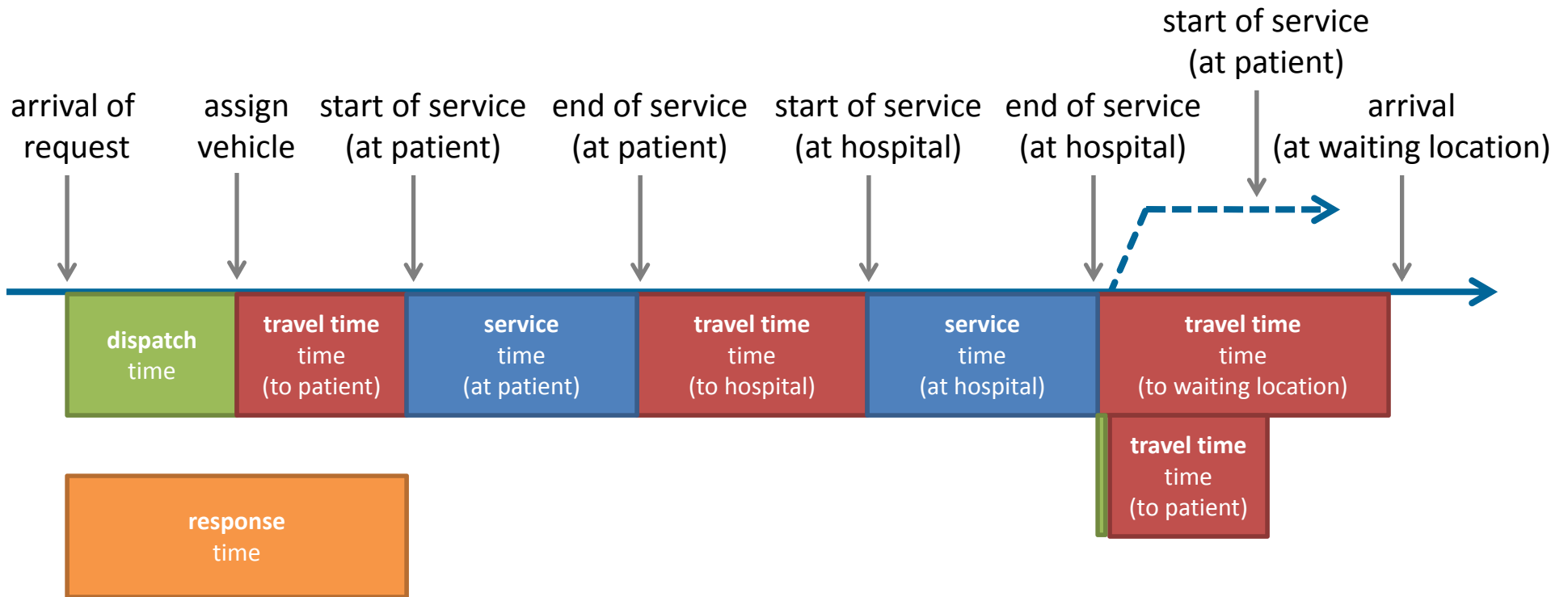


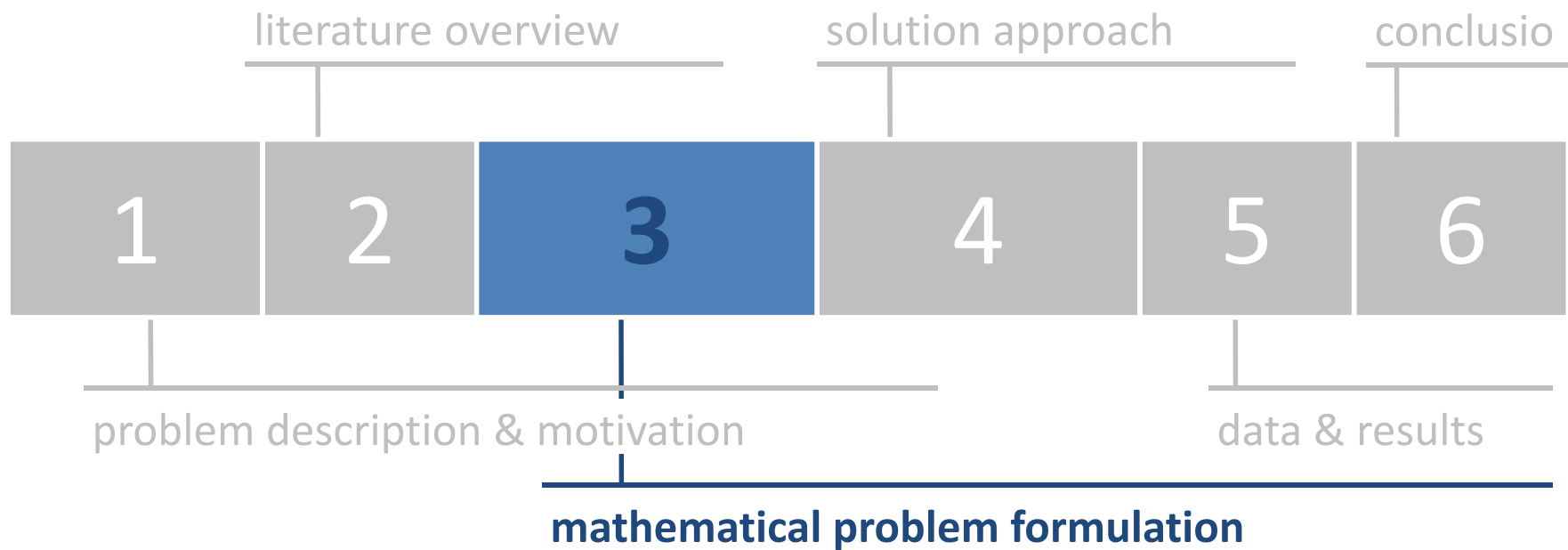
## Q4: **send** ambulances somewhere else?

- **relocation**
  - determine new waiting location
  - future response times!



# dispatching & reinsertion process





# basic notation

- **states**

- capture **current** situation (ambulances + requests)  $S_t (R_t, D_t)$

- **decisions**

- made **dynamically** over time  $x_t$
- **immediate** contribution  $C(S_t, x_t)$
- decisions have a **downstream** impact on future
- need estimate for **value** of being in a state  $V_t (S_t)$

- **sources of randomness**

- requests, durations  $W_t$

- **dynamic evolution**

$$S_{t+1} = S^M(S_t, x_t, W_{t+1})$$

# optimization

- **myopic policy**
  - optimize wrt immediate contribution

$$V_t(S_t) = \min_{x_t} C(S_t, x_t)$$

- **optimize underlying stochastic problem**

The diagram illustrates the Bellman optimality equation for a stochastic problem. The equation is 
$$V_t(S_t) = \min_{x_t} ( C(S_t, x_t) + \mathbb{E} \{ V_{t+1}(S_{t+1}(S_t, x_t, W_{t+1})) \} )$$
 The terms are annotated as follows: 

- immediate contribution**: A red box pointing to  $C(S_t, x_t)$ .
- expected future contribution**: A red box pointing to the expectation term  $\mathbb{E} \{ V_{t+1}(S_{t+1}(S_t, x_t, W_{t+1})) \}$ .
- value of next state**: A green box pointing to the  $V_{t+1}$  term.
- current state**: A green arrow pointing to  $S_t$ .
- decision**: A green arrow pointing to  $x_t$ .
- random input**: A green arrow pointing to  $W_{t+1}$ .

# state (in more detail)

- **resources (ambulances)**

- attribute vector
- resource state vector

$$a_t \in \mathcal{A}$$

$$R_t = (R_{ta})_{a \in \mathcal{A}}$$

- **demand (requests)**

- attribute vector
- demand state vector

$$b_t \in \mathcal{B}$$

$$D_t = (D_{tb})_{b \in \mathcal{B}}$$

$$S_t = (R_t, D_t)$$

- **state**

- **decision**

- elementary decisions
- decision variable

$$d \in \mathcal{D}$$

$$x_t = (x_{tad})_{a \in \mathcal{A}, d \in \mathcal{D}}$$

# states (example)

t = 10:30am

- **resource state**

- **idle** ambulances



current location

available since

- **busy** ambulances

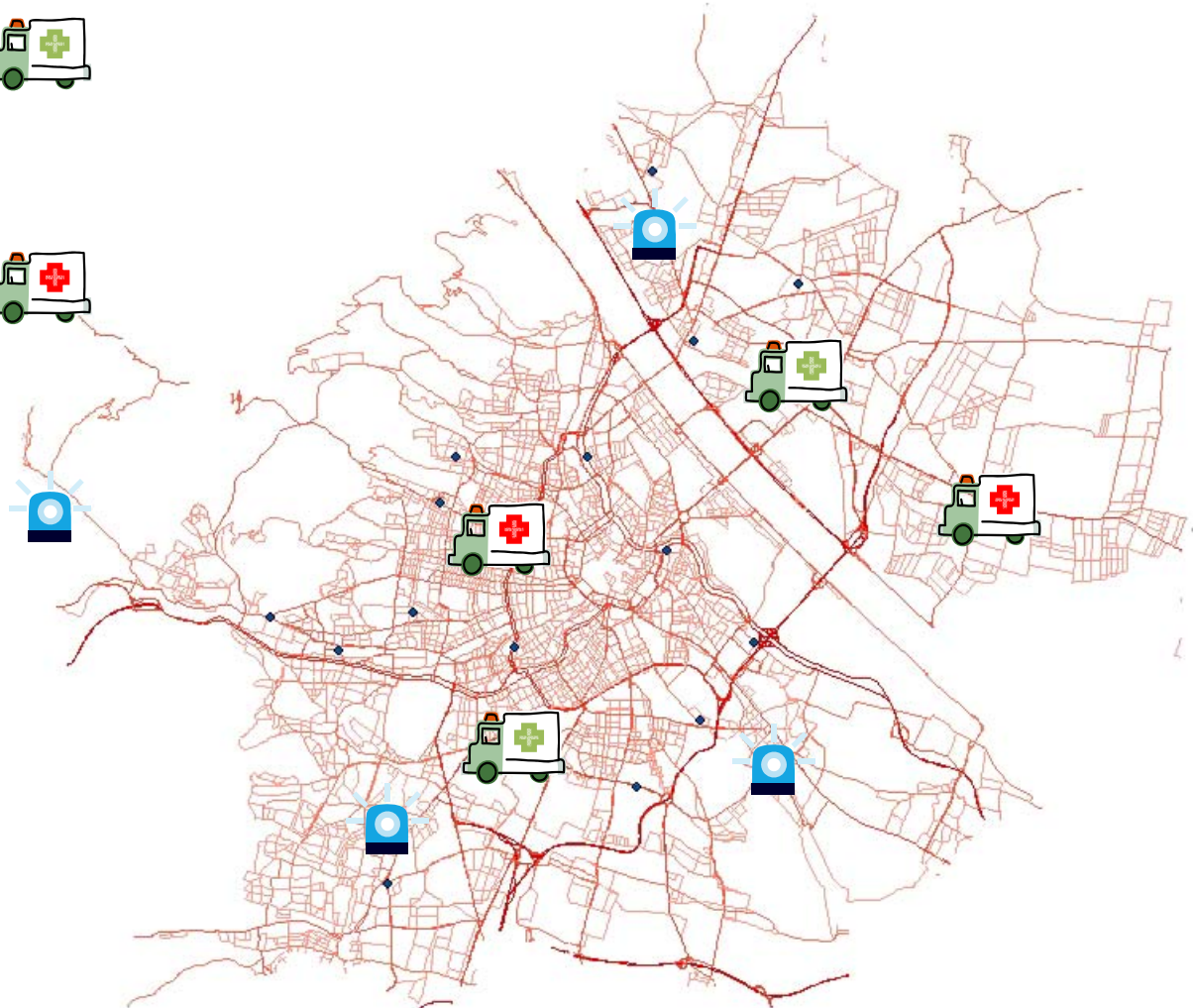


next location

available next

- **demand state**

- location
- arrival time
- priority



# model (constraints)

- **flow conservation on request**

at most one **idle** ambulance can be dispatched to any **request**

$$\sum_{a \in \mathcal{A}_t^i} x_{tad} \leq D_{tb_d} \quad \forall d \in \mathcal{D}^D$$

- **for ambulances**

only **idle** ambulances can be dispatched

$$\sum_{d \in \mathcal{D}^D} x_{tad} \leq R_{ta} \quad \forall a \in \mathcal{A}_t^i$$

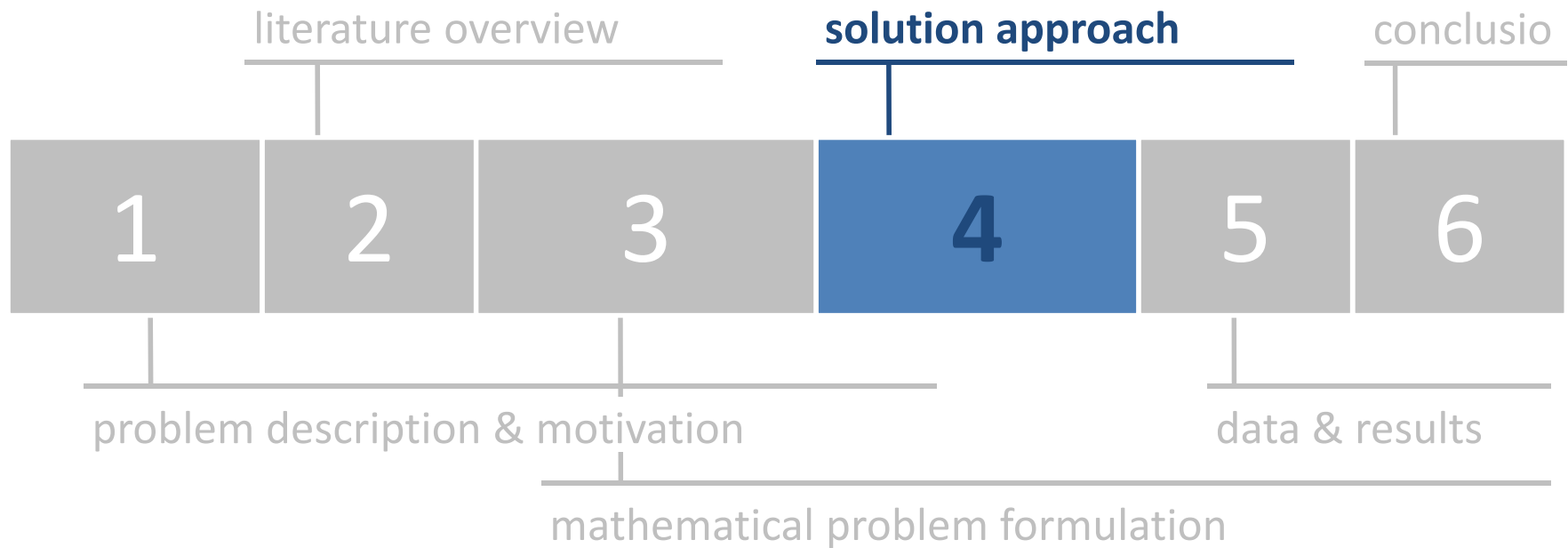
ambulances **just becoming idle** have to be dispatched or relocated

$$\sum_{d \in \mathcal{D}^D \cup \mathcal{D}^R} x_{tad} = R_{ta} \quad \forall a \in \mathcal{A}_t^{i+}$$

status of **busy** ambulances **cannot** be changed

$$\sum_{d \in \mathcal{D}} x_{tad} = 0 \quad \forall a \in \mathcal{A}_t^b$$





# dynamic programming

- **stochastic optimization problem**

$$V_t(S_t) = \min_{x_t \in \mathcal{X}_t} ( C(S_t, x_t) + \mathbb{E}\{ V_{t+1}(S_{t+1}(S_t, x_t, W_{t+1})) \} )$$

- **basic idea**

- recursive
- step backward in time

- **curses of dimensionality**

- state vector  $S_t$  grows very quickly  $|\mathcal{A}| \times |\mathcal{B}|$
- size of outcome space of random variable  $W_t$   $|\mathcal{A}| \times |\mathcal{B}|$
- size of decision vector  $x_t$   $|\mathcal{A}| \times |\mathcal{D}|$

# approximate dynamic programming (ADP)

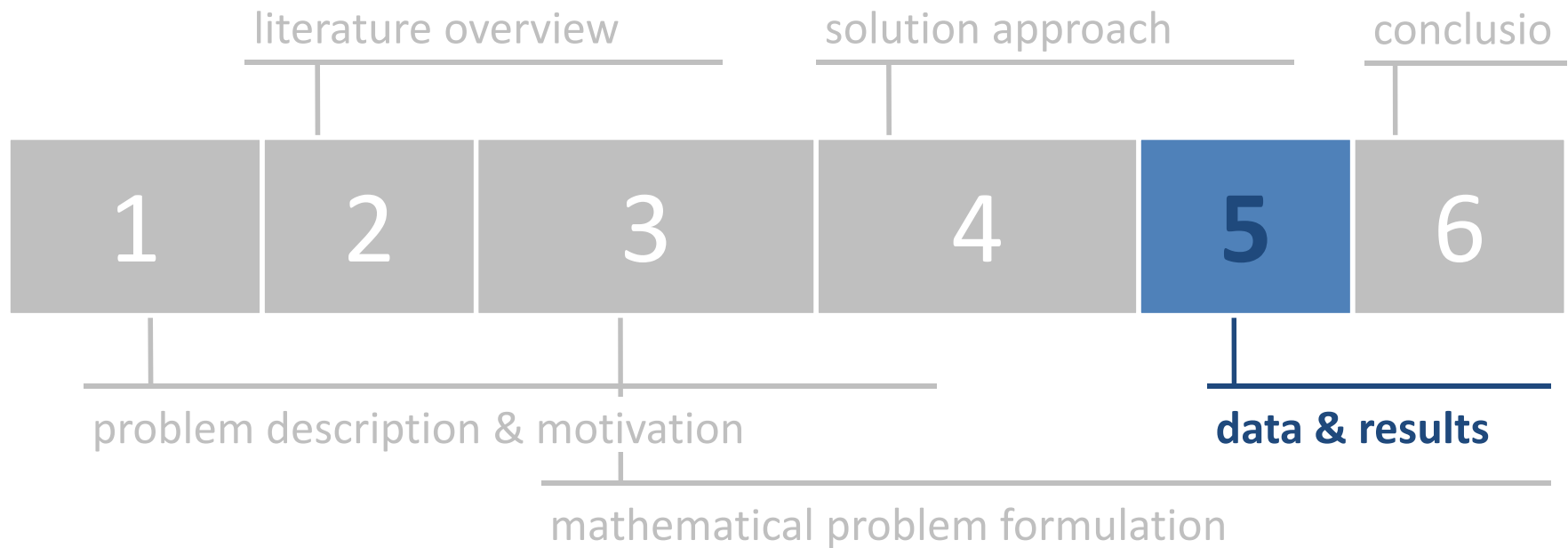
- **stochastic optimization problem**

$$\hat{v}_t^n = \min_{x_t \in \mathcal{X}_t^n} ( C(S_t^n, x_t) + \mathbb{E} \{ \bar{V}_{t+1}^{n-1} (S_{t+1}(S_t^n, x_t, \hat{\omega}_{t+1}^n)) \} )$$

- **basic idea**

- make decisions based on **approximation** of value function
- step **forward** in time (sample what might happen)
- **iteratively**. using a fresh set of sample realizations
- **update** value function approximation

$$\bar{V}_t^n(S_t) = (1 - \alpha_{n-1}) \bar{V}_t^{n-1}(S_t^n) + \alpha_{n-1} \hat{v}_t^n$$



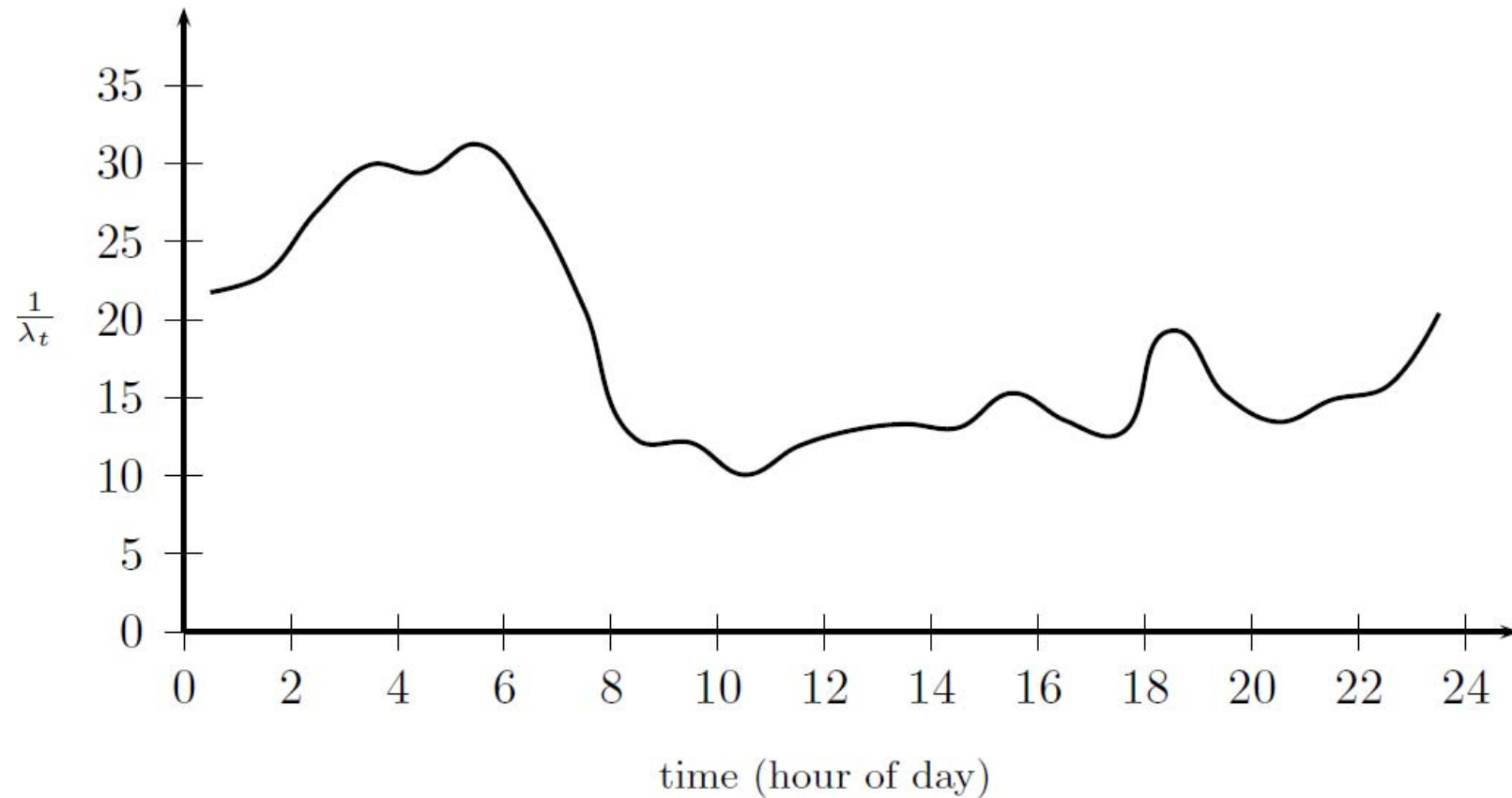
# data

- **real data**
  - street [network](#): city of Vienna (1.7 mio inhabitants, 41.5 hectare)
  - fleet of 14 ambulance vehicles, 16 locations
  - **requests**
    - average # of 89.24 emergencies per day
    - volume itself highly dependent on time of day
    - exponentially distributed [interarrival](#) times
    - spatial poisson process based on distribution of [origins](#)

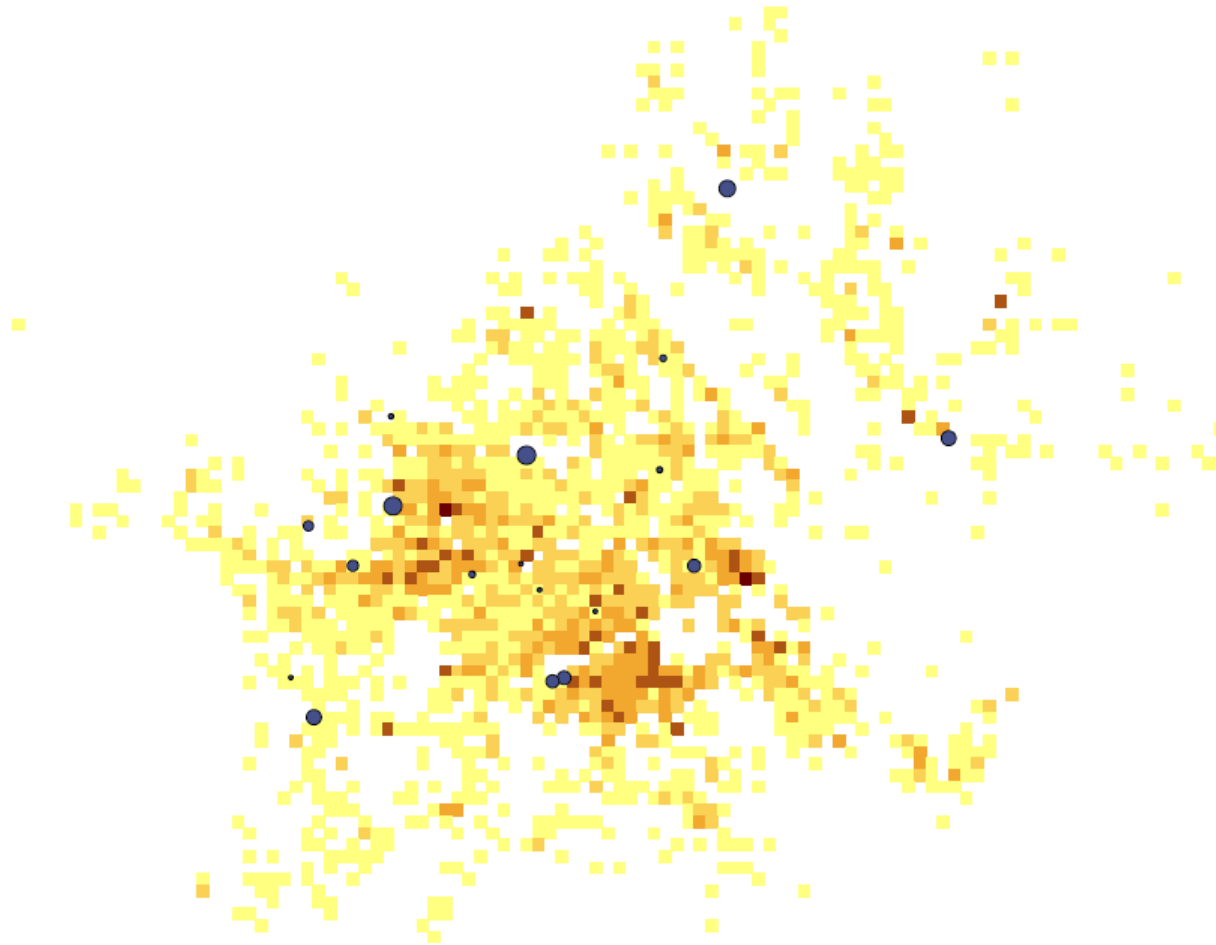
# road network & waiting locations



# interarrival times



# origin & destination of requests





# experiment setup

- **experiment setup**
  - training phase ( $10^5$  iterations)
    - fixed step size  $\alpha = 0.2$
    - temporal (spatial) aggregation parameter  $\phi_t = \phi_s = 4$
    - sampled data from estimated distributions
  - 5 independent test runs

# first training: $10^5$ iterations

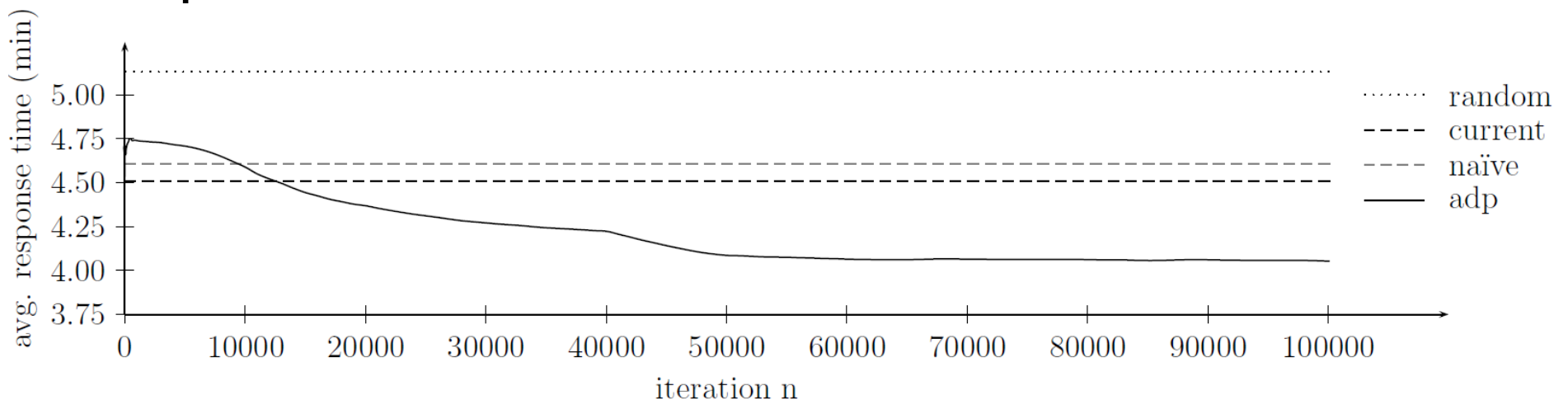
- **2 decisions**

- ~~which vehicle to dispatch~~
- **where** to relocate?

- **benchmark policies**

- relocate to **home** location 4.51 min(current strategy)
- relocate to **closest** location 4.61 min(naïve strategy)
- relocate to **random** location 5.12 min

- **adp** **4.05 min**



# relax this assumption

- **2 decisions**

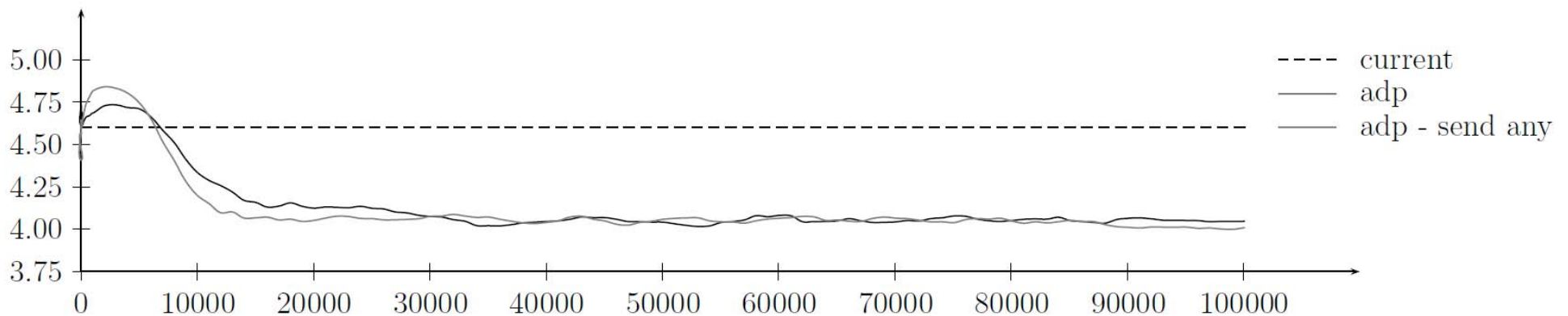
- which vehicle to dispatch (closest! regulatory rules)
- where to relocate?

- **benchmark policy**

- relocate to home location 4.60 min (current strategy)

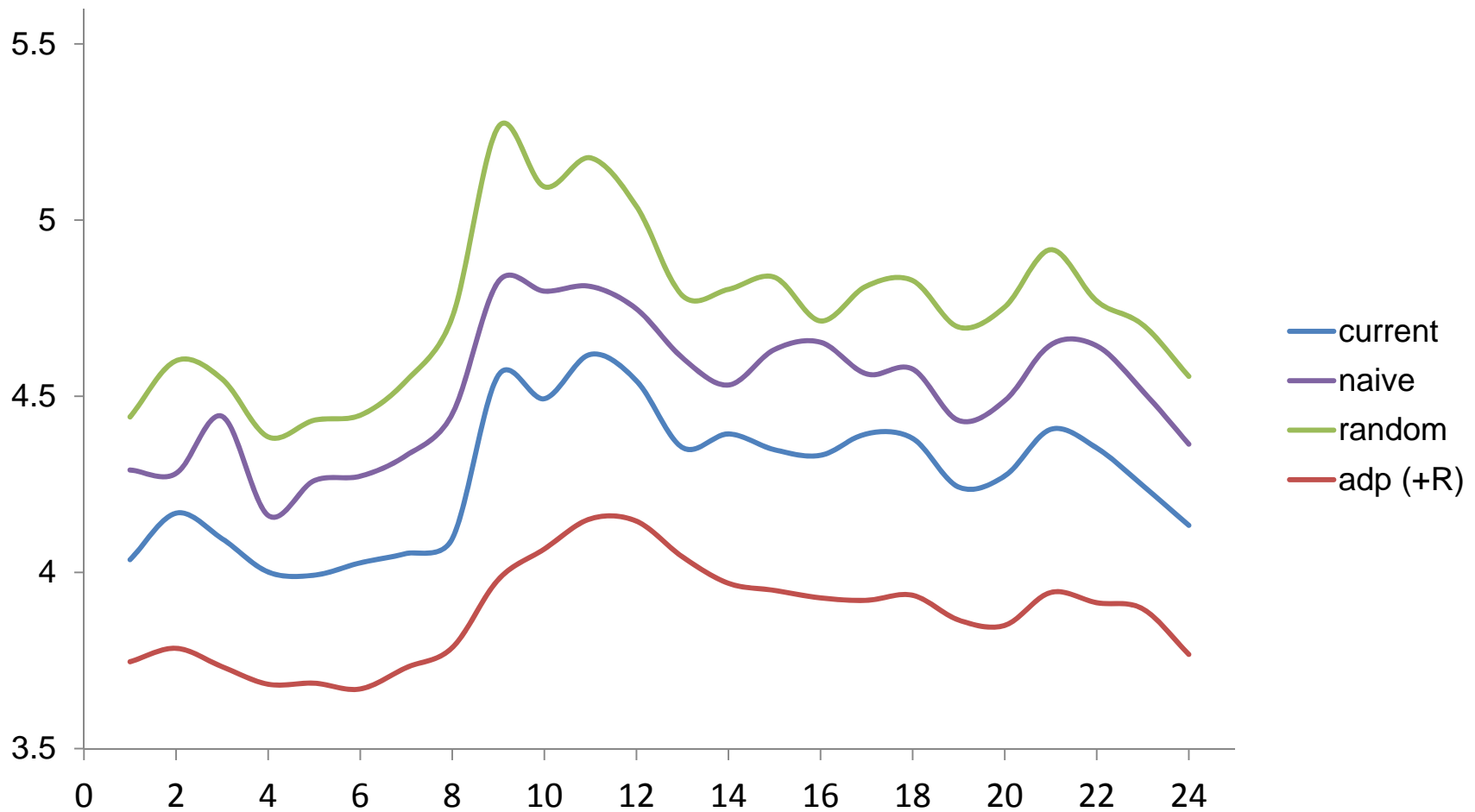
- **adp**

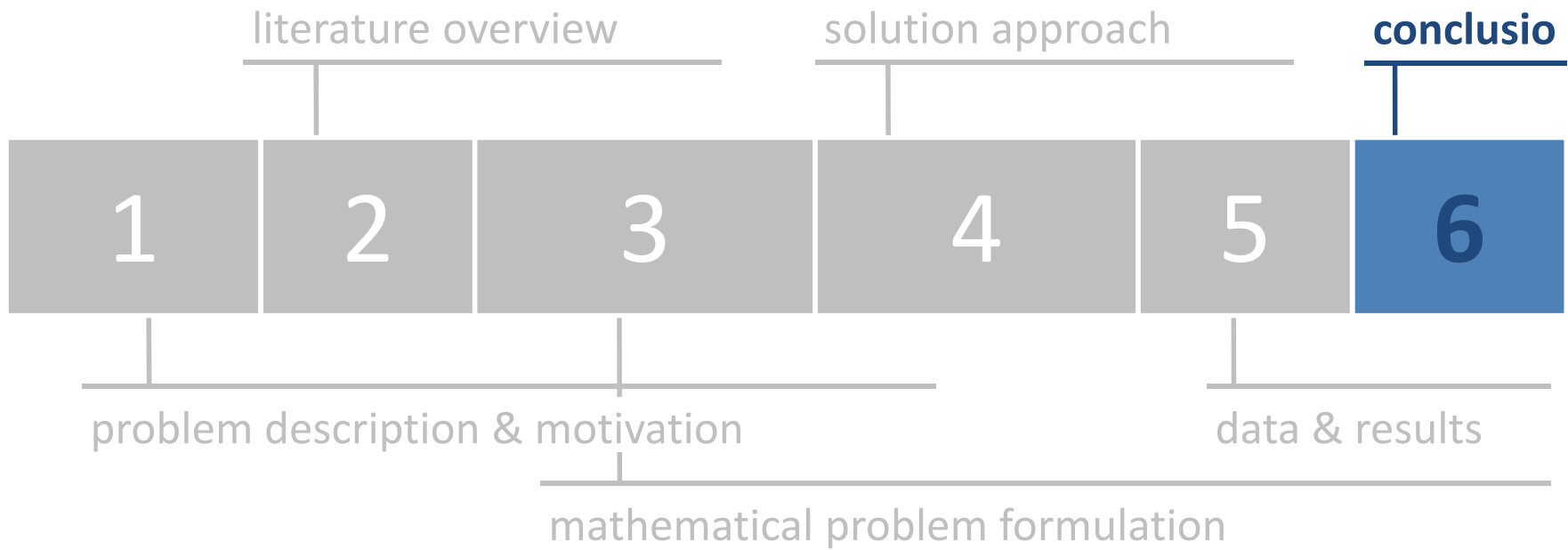
- send closest 4.05 min
- send any 4.01 min



# response times over course of day

average response time (over day)





# conclusion

- **contribution**
  - **formulated a dynamic model** for the ambulance dispatching and relocation model
  - solved using **ADP**
  - **outperformed** benchmark policies (random/naïve/current)
  - **pays off to deviate** from current dispatching rules (13%)
    - consider other vehicles (not just closest one) for **dispatching**
    - **relocate** vehicles adequately **after** finishing service
    - **relocate** vehicles **empty** to cope with current situation



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## Solving the dynamic ambulance relocation and dispatching problem using approximate dynamic programming

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### ABSTRACT

Emergency service providers are supposed to locate ambulances such that in case of emergency patients can be reached in a time-efficient manner. Two fundamental decisions and choices need to be made real-time. First of all immediately after a request emerges an appropriate vehicle needs to be dispatched and send to the requests' site. After having served a request the vehicle needs to be relocated to its next waiting location. We are going to propose a model and solve the underlying optimization problem using approximate dynamic programming (ADP), an emerging and powerful tool for solving stochastic and dynamic problems typically arising in the field of operations research. Empirical tests based on real data from the city of Vienna indicate that by deviating from the classical dispatching rules the average response time can be decreased from 4.60 to 4.01 minutes, which corresponds to an improvement of 12.89%. Furthermore we are going to show that it is essential to consider time-dependent information such as travel times and changes with respect to the request volume explicitly. Ignoring the current time and its consequences thereafter during the stage of modeling and optimization leads to suboptimal decisions.

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### 1. Introduction and related work

Emergency service providers are supposed to locate ambulances such that in case of emergency patients can be reached in a time-efficient manner. Two fundamental decisions and choices need to be made real-time. First of all immediately after a request emerges an appropriate vehicle needs to be dispatched and send to the requests' site. Ambulances, when idle, are located at designated waiting sites. Hence after having served a request the vehicle needs to be relocated (i.e. its next waiting site has to be chosen). For a close match to reality, time-dependent information for both traveling times and the request volume will be considered explicitly. We

cardiac and circulatory arrest the chances for a resuscitation to be successful decrease dramatically. Typically chances decrease by 10% per minute as long as the patient is not treated accordingly. Providing a quick response to emergency requests is crucial for the patients' state of health.

The contribution of this paper is threefold.

- (i) We propose a stochastic dynamic model for the ambulance relocation and dispatching problem, which will be solved by means of ADP.
- (ii) In order to get a preferably accurate model of reality we will explicitly take into account time-dependent information and



# Thank you for your attention!



## Questions?

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